

# An experimental method for estimating the thermal diffusivity of building elements, depending on the resolution of temperature measurement

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## Abstract

Thermal diffusivity, also known as temperature equalization coefficient, is the basic parameter in the Fourier equation for non-stationary heat exchange. Its values are known for homogeneous materials with a specific composition. Building elements made of reinforced concrete, for example, have a heterogeneous structure. For such cases, table values from the literature may differ significantly from the specific object for in real constructions. More accurate thermal diffusivity values can be obtained from measurements for a given element. Since these are usually large sized elements, the measurement method should take into account the material in the entire volume of the element. Proposals for such a method based solely on temperature measurement at several depths in the sample were presented. It consists in solving the inverse problem assuming a polynomial solution of the Fourier equation. An attempt was made to validate the method through a numerical experiment. Temperature variability was simulated with one-dimensional flow in the wall with assumed thermal diffusivity. Then the value of this diffusivity was determined from the calculated temperatures. On the inside of the partition, a constant temperature was maintained and on the outside it changed periodically. The dependence of the error in the obtained diffusivity value on the precision of temperature results was analyzed. Depending on the precision of the calculations, a minimum relative error of 2 to 6 percent was obtained. With the help of the data presented in the article, conclusions can be drawn as to the conditions that must be met to determine the value of diffusivity in real measurements with the required accuracy. The obtained results indicate that this method is worth further research.

**Keywords:** thermal diffusivity, physical properties

## 1 Introduction

Measurement of thermal diffusivity for a given material can be a reliable way to determine its value in calculations for a specific case. This is particularly important in building elements made of reinforced concrete, for example, because they have a heterogeneous structure. Values obtained from literature may differ significantly from the specific case for which we model heat transfer. Since these are usually large sized elements, the average value for the simulated element should be taken into account. The proposed method of measuring thermal diffusivity is based solely on reading the temperature at several depths in the sample in the assumed period of time. It consists in solving the inverse problem assuming a polynomial solution of the Fourier equation. The very idea of determining diffusivity based on temperature measurement has already been proposed, for example in [1, 2, 4], but the method of interpreting these measurements is new. Most of the methods used are based on matching the coefficients in the ready solution of the Fourier equation for certain boundary conditions. The necessary condition for obtaining the correct value is one-dimensional heat flow in the sample. The subject of the article is method validation through a numerical experiment. The temperature variability in the wall with the assumed thermal diffusivity was simulated. Boundary conditions were selected to obtain one-dimensional heat flow. Then the value of this diffusivity was determined from the calculated temperatures. The boundary conditions corresponded to the outer wall of the building. On the inside of the partition, a constant temperature was maintained and on the outside it changed periodically. The dependence of the error in the obtained value on the precision of the calculated temperature values was analyzed.

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## 2 Mathematical model

With one-dimensional heat flow, for example in the outer wall of a building, the temperature distribution satisfies the Fourier equation for heat conduction [3, 6].

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2} \quad (1)$$

Where  $a$  is thermal diffusivity given by the equation:

$$a = \frac{\lambda}{\rho c_p} \quad (2)$$

Suppose that a function in the form of a polynomial of two variables is fulfills the equation (1). In the first variant, that was assumed for this function is in the form of:

$$T(c, t) = p00 + p10x + p01t + p20x^2 + p11xt + p02t^2 \quad (3)$$

where  $x$  is the coordinate along the wall thickness and  $t$  time.

By transforming equation (1) the relationship into thermal diffusivity can be written as

$$a = \frac{\frac{\partial T}{\partial t}}{\frac{\partial^2 T}{\partial x^2}} \quad (4)$$

Using the function (3) it is possible to determine the numerator and denominator of equation (4) by calculating the appropriate derivatives. After substitution it is obtained:

$$a = \frac{p01 + p11x + 2p02t}{2p20} \quad (5)$$

The values of the coefficients  $p00$ ,  $p10$ ,  $p01$ ,  $p20$ ,  $p11$  and  $p02$  are obtained from the approximation of the temperature values in the tested element. The received function will probably not be an exact solution (1) but with some error. In formula (5) the value of diffusivity is a function of time and space. Therefore, there is a problem of interpretation of these changes, it was assumed that we will determine the average value in the time and spatial range for a given measurement. In the second variant, it was assumed that the solution (1) is approximated by:

$$T(x, y) = p00 + p10x + p01t + p20x^2 \quad (6)$$

what means:

$$a = \frac{p01}{2p20} \quad (7)$$

and the determined diffusivity value is unambiguous. It is possible to approximate functions at different lengths of time and to measure temperature with different resolutions. The subject of this article is to examine the impact of temperature reading precision on the calculated value of thermal diffusivity.

### 3 Geometry and boundary conditions in simulation

In the numerical experiment, a wall thickness of 0.3 meters was assumed with a specific heat of 800 J/(kgK), thermal conductivity of 1 W/(mK) and a density of 2000 kg/m<sup>3</sup>, for these values we obtain the thermal diffusivity of the wall

$$a = \frac{\lambda}{\rho c_p} = \frac{1}{800 \cdot 2000} = 6,25e-7 \frac{m^2}{s} \tag{8}$$

Equation (1) was solved numerically using the finite volume method [5] in geometry as in Figure 1

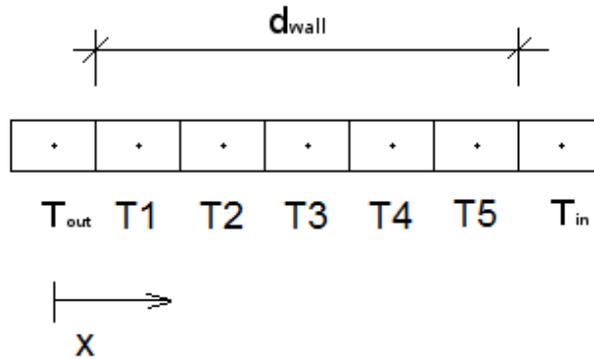


Figure 1. Geometry and boundary conditions in the wall

One-dimensional heat conduction was assumed,  $T_{in}$  temperature is constant at 20°C from the inside of the wall. The control volume grid is one-dimensional. From the outside, the temperature changes with the period of 24h as shown in Figure 2, it is a simulation of the daily temperature variation over the summer. It was assumed that no external heat sources will be used, only natural conditions. This simplifies the method although it makes the accuracy of the results obtained dependent on daily temperature courses.

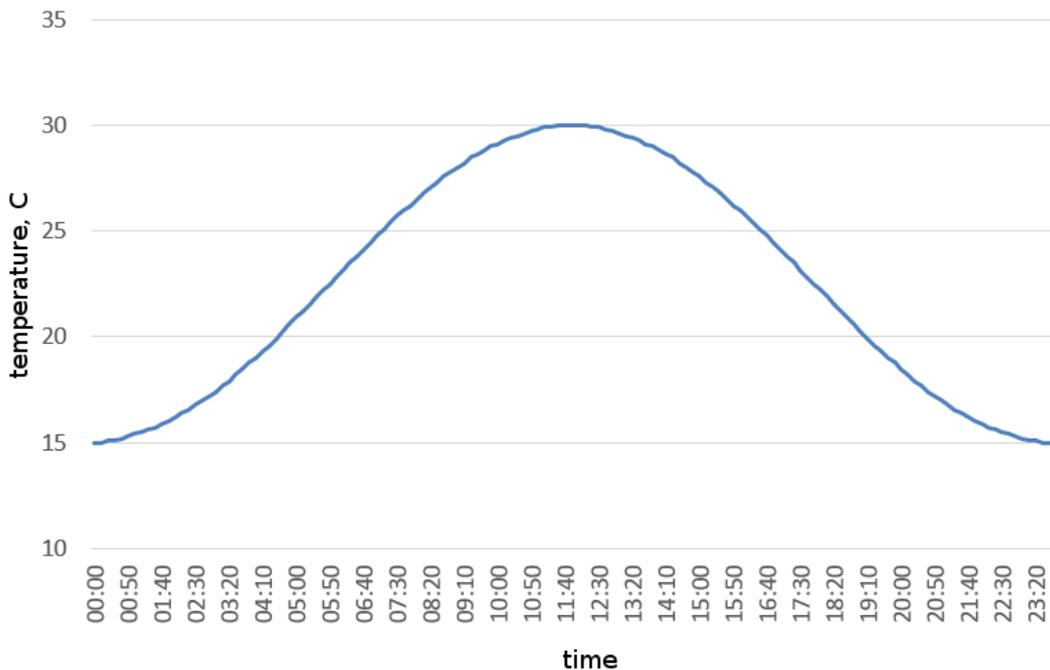


Figure 2. Outside temperature used for calculation

At the beginning of the simulation, the temperature of the entire wall was 20°C. The temperature field was simulated for five days in order to obtain reproducible variability throughout the geometry. The temperature in nodes

on the fifth day is shown in Figure 3. The results were recorded at intervals of 10 minutes.

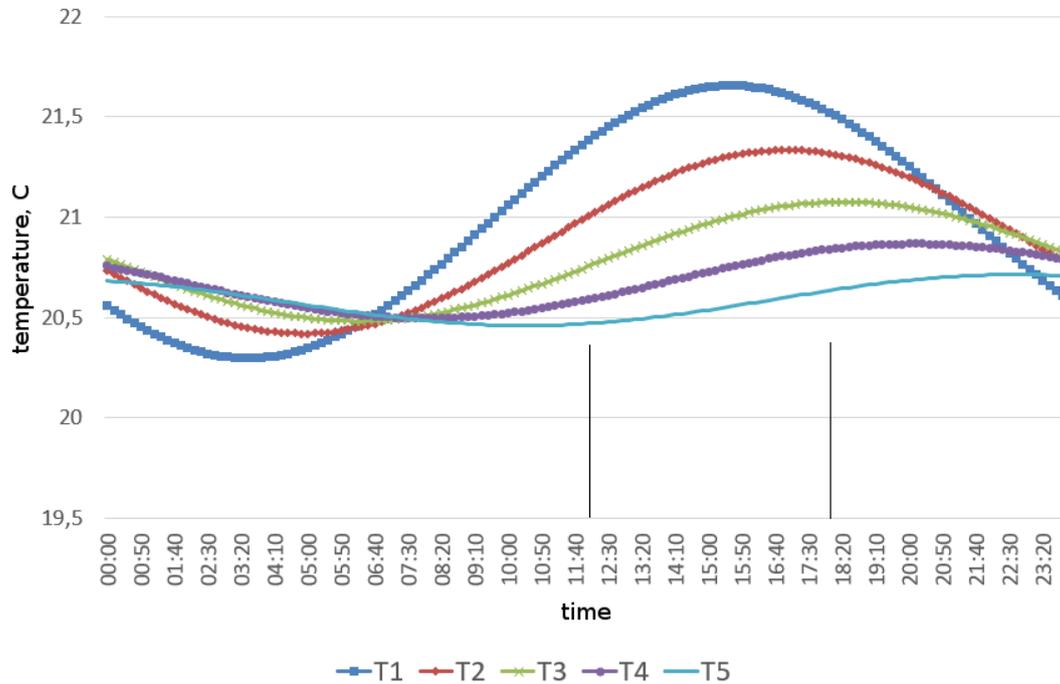


Figure 3. The temperature inside the wall in fifth day of simulation

Thermal diffusivity was calculated on the basis of temperature in the range marked in figure 3 by vertical lines. The choice of range was subjective, guided mainly by the fact that the temperature waveforms do not overlap. This interval was divided into sub-ranges containing temperatures in nodes T1-T5 for seven consecutive recorded results, i.e. sixty minutes. Thermal diffusivity was determined for each of these sub-compartments separately. The algorithm of the procedure consisted in approximating data from the sub-compartment in turn using functions (3) and (6) and then determining the diffusivity from (5) and (7), respectively. Calculations were made for the temperature obtained with the accuracy of four and one decimal place.

## 4 Results

The diffusivity values for each sub-compartment were obtained. They differed depending on the time from which the data came. This proves that there is a problem of choosing the right time to measure the temperature. The results for the approximation by the function (3) for a different number of decimal places are shown in Figure 4. The exact value of diffusivity is also indicated in the figure.

The results for the approximation by the function (6) also for a different number of decimal places are shown in Figure 5.

## 5 Conclusions

The results obtained for approximating the solution by a function of a higher degree and high measurement precision show a relative error of about 2%. The condition is the proper selection of the measurement time. For the accepted range it is its beginning of a period. The error continues to increase. When reading the temperature with an accuracy of one decimal place, the diffusivity value oscillates around the value calculated with high measurement precision. Similar oscillations were obtained in [4] and [7] for thermal resistance. The average diffusivity value for low measurement precision was calculated from the first eight measurements. The value 5.71E-07 was obtained, which gives a relative error of 8.69%.

For the approximation by function (6), the measurement uncertainty is greater. For some time, about half of the analyzed time, however, we also get results close to the exact value.

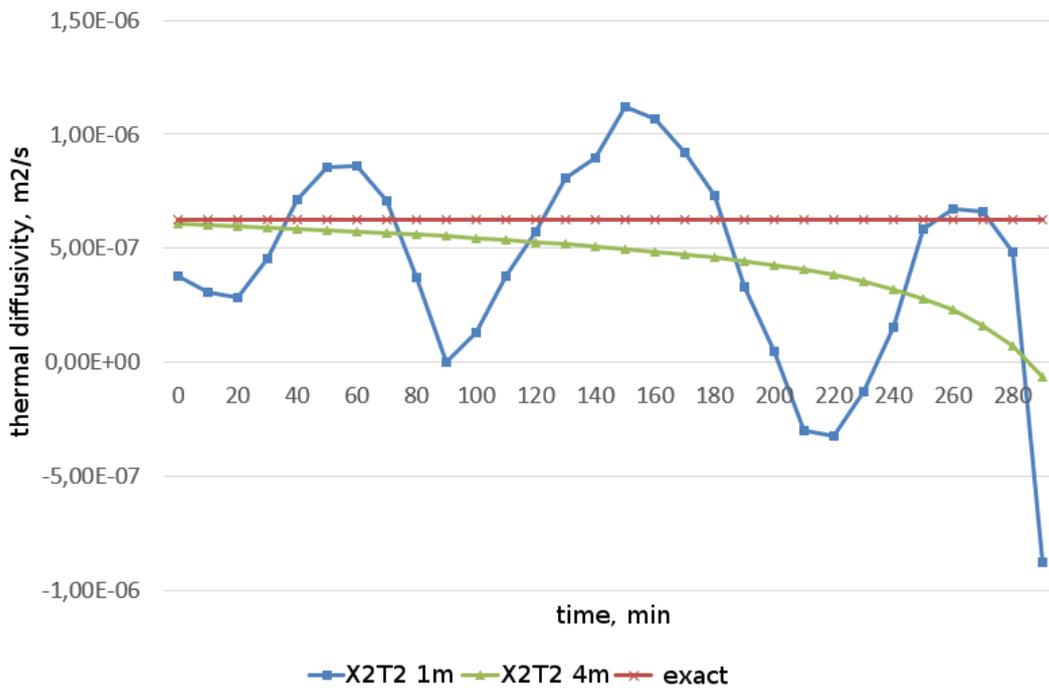


Figure 4. The diffusivity values obtained with the approximation of measurements are the square function with respect to time and x coordinate (X2T2)

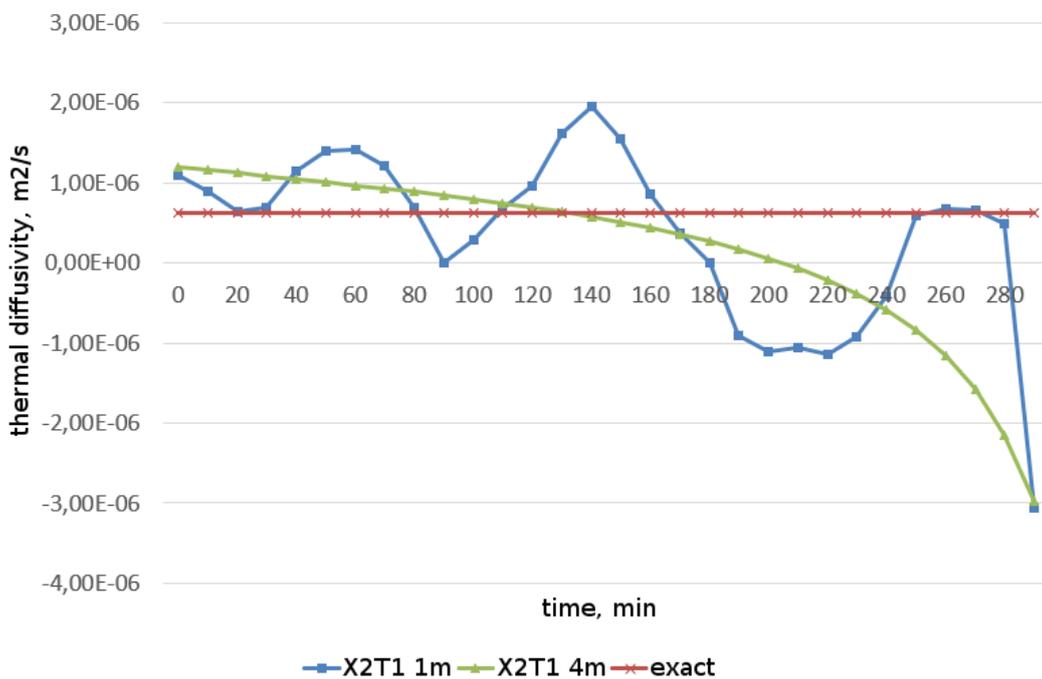


Figure 5. The diffusivity values obtained with the approximation of measurements are a linear function versus time and a quadratic function relative to the x coordinate (X2T1)

The obtained diffusivity value, and therefore also the error is variable in time, using the data presented in the article, one can draw conclusions as to the conditions that must be met to determine the value of diffusivity in real measurements with the required accuracy. Based on the comparison of Figures 3 and 4, it can be concluded that for high precision of measurement and function of the second stage of time and space, the results deteriorate after passing

the daily maximum temperature. The obtained results indicate that this method is worth further research.

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